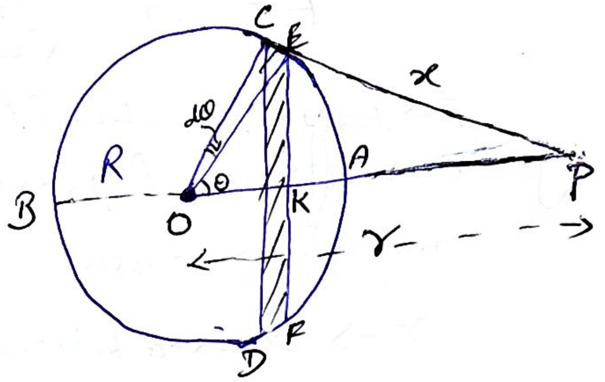
B.Sc Physics Part-I
Paper-I, Group-B

## Gravitational Potential Due to a Spherical Shell:

(i) Potential at a point outside the shell:-

We have taken a spherical shell of radius $R$. (See Fig.)




Let the surface density $= \sigma$ (mass per unit area of surface)

→ P is the point ~~located at~~ a distance $r$ from the centre of shell O.

i.e. $OP = r$.

We cut out the slice CEFD (denoted by the shaded region in fig.)

CEFD → slice in the form of a ring by two planes CD and EF and perpendicular to OP.

Let $\angle EOP = \theta$ and $\angle COE = d\theta$

The radius of the ring is $EK = OE \sin\theta = R\sin\theta$

and Circumference $= 2\pi R\sin\theta$,

width $= CE = R \cdot d\theta$.

Thus, the surface area of the ring $= 2\pi R\sin\theta \times R\,d\theta$

and therefore, mass $= 2\pi R\sin\theta\, R\,d\theta \cdot \sigma$

$= 2\pi R^2 \sin\theta\, d\theta \cdot \sigma$

Let $EP = x$; therefore potential at point P due to the ring $dV = -G \frac{\text{mass of slice}}{x} = -G \frac{2\pi R^2 \sin\theta\, d\theta\, \sigma}{x}$ —— (1)

Next, in $\triangle OEP$, $EP^2 = OE^2 + OP^2 - 2OE\,OP\cos\theta$

or $x^2 = R^2 + r^2 - 2Rr\cos\theta$

Differentiating above expression

$$2x\,dx = 0 + 0 + 2Rr\sin\theta\, d\theta$$

or $$x = \frac{Rr\sin\theta\, d\theta}{dx}$$

Thus, eqn (1) can be written as

$$dV = -G \frac{2\pi R^2 \sin\theta\, d\theta\, \sigma}{Rr\sin\theta\, d\theta} \cdot dx$$

or $$dV = -G \frac{2\pi R\sigma}{r} dx$$ —— (2)

Now integrating eqn (2)

we take x limits from $x = AP = r-R$ to $x = BP = r+R$.

$$V = -\int_{r-R}^{r+R} G \cdot \frac{2\pi R\sigma}{r} dx = -\frac{G 2\pi R\sigma}{r} \int_{r-R}^{r+R} dx$$

or $$V = -G \frac{2\pi R\sigma}{r} [x]_{r-R}^{r+R} = -\frac{2\pi R\sigma G}{r} \cdot 2R$$

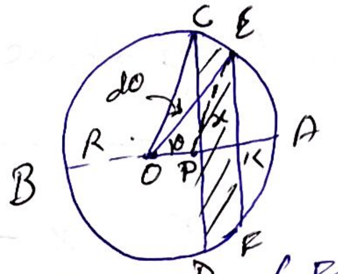
or $$V = -\frac{4\pi R^2 \sigma G}{r}$$

$4\pi R^2 \sigma =$ m (mass of the shell)

Thus, $\boxed{V = -G\frac{m}{r}}$ — ③

(ii) Potential at a point on the surface of the shell:

Let point P be on the surface of the shell i.e. Consider point P at A.



(Fig-2)

(Fig for Potential inside the shell)

To obtain the potential, we again need to integrate Eq. ②.

But for the present case, limit goes from $x = PA = 0$ to $x = PB = 2R$

$$\therefore \quad V = \int_0^{2R} -G\cdot\frac{2\pi R\sigma}{r}\,dx = \frac{-2\pi R\sigma G}{r}\times 2R$$

$$\text{or} \quad V = -\frac{4\pi R^2\sigma G}{r}$$

$$\text{or} \quad \boxed{V = -\frac{mG}{R}} \text{ — ④} \quad \{\text{here } r = R\}$$

(iii) Potential at a point inside the shell:

Integrating eq. ② by taking x limits from $x = PA = R-r$ to $x = PB = R+r$, we get $\boxed{V = -\frac{GM}{R}}$

since $V = \int_{R-r}^{R+r} -G\frac{2\pi R\sigma}{r}\,dx = -\frac{2\pi R\sigma G}{r}\cdot 2r = -4\pi R\sigma G$

(see Fig-2)

$= \frac{-4\pi R^2\sigma G}{R} = -\frac{GM}{R}.$